GCSE Maths – Geometry and Measures

Pythagoras' Theorem

Notes

WORKSHEET

Pythagoras' Theorem

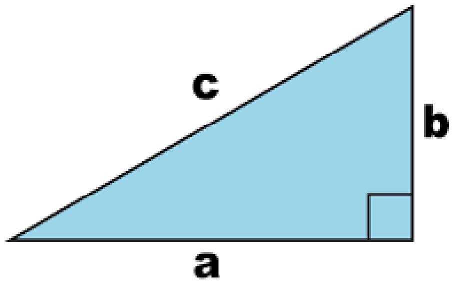
$$a^2 + b^2 = c^2$$

Pythagoras' Theorem is a formula which relates the side lengths of a **right-angled triangle**.

The theorem states that the square of the hypotenuse length is equal to the sum of the squares of the other two lengths. Using the formula $a^2 + b^2 = c^2$ we can calculate a side length of the triangle, when the other two lengths are known.

Labelling the right-angled triangle

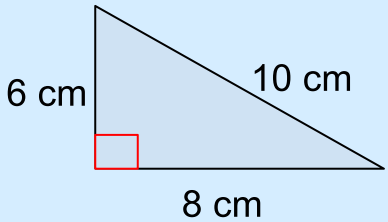
To use this theorem, we must be able to label the lengths **a**, **b**, and **c** correctly on the triangle.



Length "c" is always the hypotenuse of the right-angled triangle.

The **hypotenuse length** is located **opposite the right angle** and is the **longest side** of the triangle. Length **"a"** and **"b"** are the other two side lengths of the triangle. They can be labelled in any order.

Example: In the triangle below, what is the length of the hypotenuse?



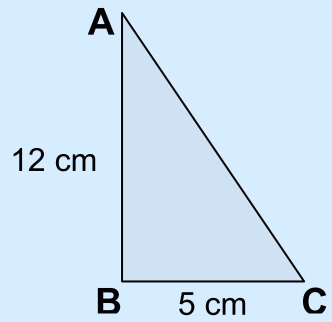
The hypotenuse is the length opposite to the right angle in this triangle.

The length opposite to the right angle is 10 cm.
*Therefore, the length of the hypotenuse is **10 cm**.*

Finding the length of the hypotenuse

The length of the hypotenuse can be found by using the formula $\boldsymbol{a^2 + b^2 = c^2}$ and substituting the values for $\boldsymbol{a}$ and $\boldsymbol{b}$.

Example: Calculate the length of AC in the triangle below



1. Use the formula and **substitute** in values of a and b.

$$\boldsymbol{c^2 = a^2 + b^2}$$
$$AC^2 = 5^2 + 12^2 = 25 + 144 = 169$$

2. **Calculate** length AC by **square rooting** each side.

$$AC = \sqrt{169}$$
$$AC = \textbf{13 cm}$$

Finding the Length of the Shorter sides

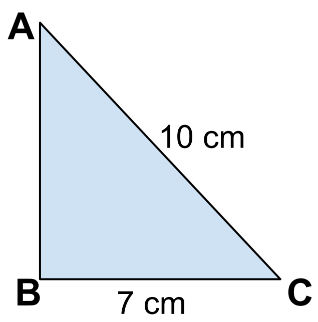
To calculate the other side length, we must **rearrange the equation**. As both **"a"** and **"b"** can represent any of the shorter sides, the equation must be rearranged to isolate **"a"** or **"b"**.

$$\boldsymbol{a^2 + b^2 = c^2}$$

Subtracting $\boldsymbol{a^2}$ from both sides:

$$\boldsymbol{b^2 = c^2 - a^2}$$

Example: Calculate the length of AB in the triangle below (to 1 decimal place)



1. Use the formula and **substitute** in values of a and b.

$$\boldsymbol{a^2 + b^2 = c^2}$$
$$a^2 = c^2 - b^2$$
$$AB^2 = 10^2 - 7^2$$
$$AB^2 = 100 - 49 = 51$$

2. **Calculate** the length AB by square rooting each side.

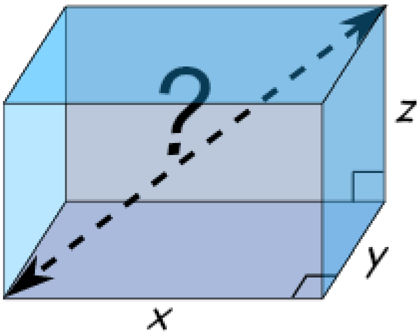
$$AB = \sqrt{51}$$
$$AB = 7.141 \ldots \text{ cm} = 7.1 \text{ cm (1 d.p.)}$$

Pythagoras in 3D (Higher Only)

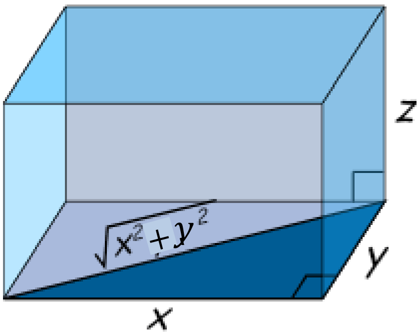
Pythagoras' Theorem can be applied to **3D shapes** to calculate heights, lengths of sides and lengths of diagonals.

Cuboids

In cubes and cuboids, we can use Pythagoras' Theorem to calculate the length of the diagonal and another formula can be generated from this.

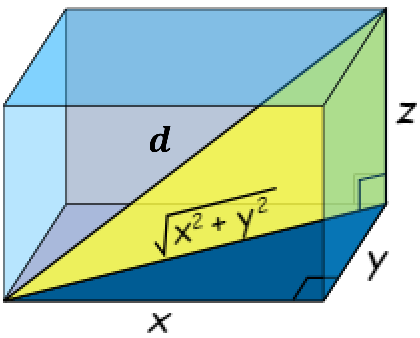


To find the length of the diagonal, we can split the cuboid up into 2 different right-angled triangles:



***First**, Pythagoras' Theorem can be used to calculate the diagonal, c, across the bottom.*

$$a^2 + b^2 = c^2$$
$$x^2 + y^2 = c^2$$
$$\sqrt{x^2 + y^2} = \mathrm{c}$$



Second, use the value calculated above to calculate the length of the diagonal of the cuboid.

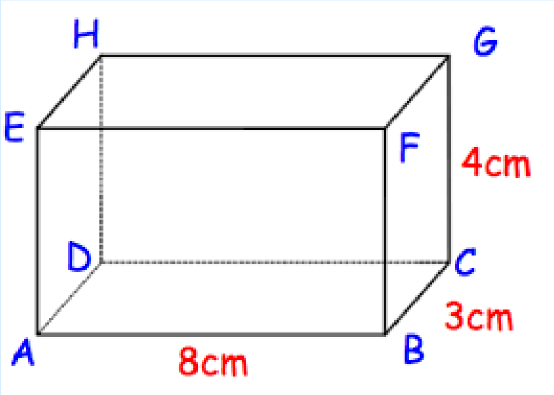
$$a^2 + b^2 = d^2$$
$$\left(\sqrt{x^2 + y^2}\right)^2 + z^2 = d^2$$
$$x^2 + y^2 + z^2 = d^2$$
$$\sqrt{x^2 + y^2 + z^2} = d$$

Therefore, the length of the **diagonal** is $\sqrt{\boldsymbol{x^2 + y^2 + z^2}}$.

You can use the previous ideas and formulas to find the **diagonals of a cuboid**.

Although you can substitute into the formula stated above, it's often good practice to **reconstruct** the formula by first finding the length of the diagonal of the base rectangle and then using this value to find the diagonal value of the cuboid. This process involves using **Pythagoras' Theorem twice**.

Example: Calculate the length of the diagonal AG in the cuboid below (to 1 decimal place)



1. Use the formula and **substitute** in values of x, y and z.

$$\boldsymbol{d^2 = x^2 + y^2 + z^2}$$

$$AG^2 = AB^2 + BC^2 + CG^2$$

$$AG^2 = 8^2 + 3^2 + 4^2$$

2. **Evaluate** the left-hand side of the equation.

$$AG^2 = 64 + 9 + 16$$

$$AG^2 = 89$$

3. **Calculate** the length AG by square rooting each side.

$$AG = \sqrt{89}$$

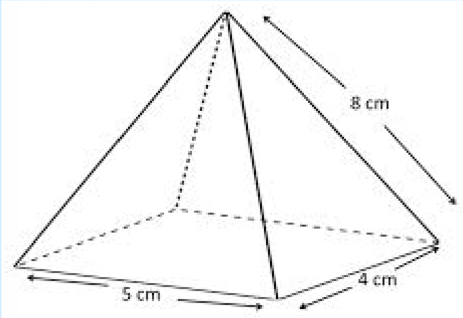
$$AG = 9.433 \ldots \text{ cm}$$

$$AG = \mathbf{9.4 \text{ cm}} \text{ (1 d.p.)}$$

Pyramids

We can use Pythagoras' Theorem to calculate lengths of a pyramid, including the height.

Example: Calculate the height in the pyramid below (to 1 decimal place)



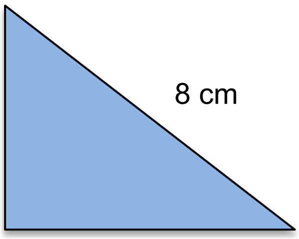
1. To **calculate** the height, first the diagonal of the base should be calculated.

$$\boldsymbol{c^2 = a^2 + b^2}$$

$$c = \sqrt{5^2 + 4^2} = \sqrt{25 + 16} = \sqrt{41}$$

2. Construct a new right-angled triangle which will allow you to find the height.

*The height is **perpendicular** to the diagonal and meets the diagonal at its **midpoint***



The base length of the new triangle is $\frac{\sqrt{41}}{2}$ **cm** *because it is half of the length of the diagonal found in step 1.*

3. Using the triangle drawn above, use the Pythagoras' Theorem to calculate the height.

$$\boldsymbol{a^2 = c^2 - b^2}$$

$$H^2 = 8^2 - \left(\frac{\sqrt{41}}{2}\right)^2 = 64 - \frac{41}{4}$$

$$H^2 = 53.75$$

4. Calculate the height by square rooting each side.

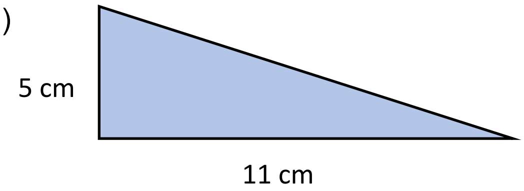
$$H = \sqrt{53.75}$$

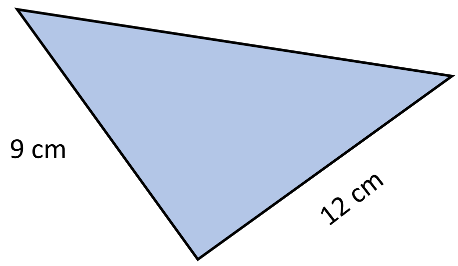
$$H = 7.332\ldots \text{cm}$$

$$\mathbf{H = 7.3\ cm}\ (1\ d.p)$$

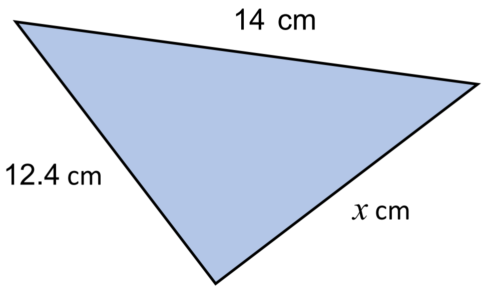
Pythagoras' Theorem – Practice Questions

1. Calculate the length of the hypotenuse, giving your answer to 3 significant figures.

a)


b)


2. Calculate the length x, giving your answer to 3 significant figures.

a)


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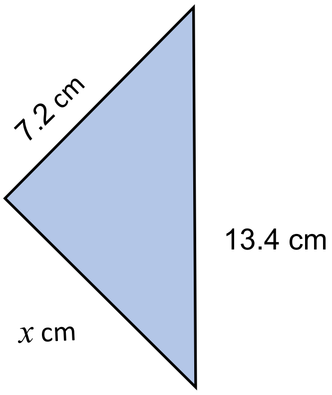
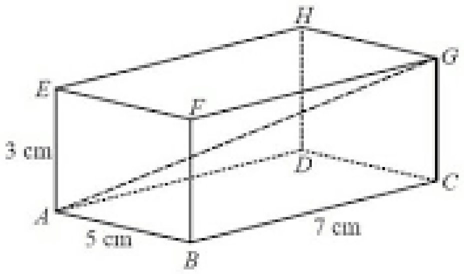
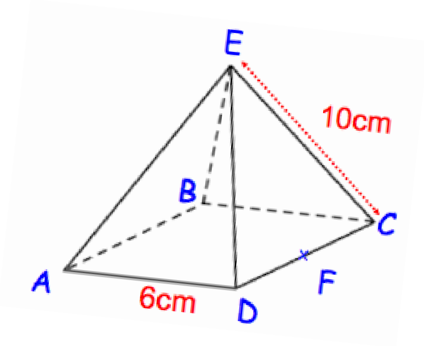


3. Calculate length AG (1 decimal place)



4. Calculate the height in this square-based pyramid (1 decimal place)



Worked solutions for the practice questions can be found amongst the worked solutions for the corresponding worksheet file.